Master of Science Program in Mathematics

July 2015

Indian Institute of Technology Jodhpur
Master of Science (M.Sc.) Program in Mathematics
Curriculum

<table>
<thead>
<tr>
<th>Cat.</th>
<th>Course Number</th>
<th>Course Title</th>
<th>L-T-P</th>
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<tr>
<td>C</td>
<td>MA511</td>
<td>Linear algebra</td>
<td>3-0-0</td>
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<td>C</td>
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<td>MA524</td>
<td>Numerical Analysis</td>
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<td>MA515</td>
<td>Programming techniques</td>
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Electives

<table>
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Note: The students are encouraged to opt electives from B.Tech., M.Tech. and PhD. level courses having mathematical orientation; e.g. Cryptography, Information Theory and Coding, Data Structures and Algorithms, Digital Image Analysis, etc. This will assist in integrating technological aspects into mathematics education.
Course Title: Linear Algebra
Course No.: MA511
Department: Mathematics
Structure (LTPC): 3-0-0 [3]
Offered for: M.Sc. Students
Type: Compulsory
Pre-requisite: None
To take effect from:

Objectives
To give sufficient knowledge of the subject, which can be used by student for further applications in their respective domains of interest.

Learning Outcomes
1. Concept of linear spaces, mapping between spaces, norm and their action on spaces.
2. Triangularization, diagonalization and Primary decomposition theorem.
4. Bilinear forms and Tensor products

Contents
1. Vector Spaces over fields, subspaces, bases and dimension. Direct sum of the subspaces, System of linear equations, Matrices and rank
2. Linear Transformations, Rank and Nullity theorem, Representation of linear transformations by matrices, duality and transpose.
3. Inner product spaces, Gram-Schmidt orthonormalization, orthogonal projections, linear functionals and adjoints, Hermitian, self-adjoint, Unitary and normal operators, Spectral theorem for normal operators, Rayleigh quotient, Min-Max principle
4. Eigenvalues, Eigenvectors, Characteristic polynomials, minimal polynomials, Cayley Hamilton Theorem, triangulation, diagonalization, Jordan canonical forms, Bilinear forms, symmetric and skew-symmetric bilinear forms, positive definiteness
5. Applications of linear algebra

Reference Books
Course Title: Real Analysis  
Course No.: MA512  
Department: Mathematics  
Offered for: M.Sc. Students  
Pre-requisite: None  
Type: Compulsory

Objectives
1. To train the student in the area of real analysis
2. To give sufficient knowledge of the subject which can be used by student for further applications in their respective domains of interest

Learning Outcomes
1. Metric spaces, compactness, completeness and connectedness
2. Measurability, integrability, Monotone convergence theorem
3. Functions of several variables, inverse function theorem and implicit function theorem

Contents
1. Functions, relations, countable and uncountable sets, axiom of choice, Zorn’s lemma,  
3. Functions of several variables, continuity, differentiability, partial derivatives, Jacobian, Inverse Function Theorem and Implicit Function Theorem  
4. Applications of real analysis

Reference Books
Objectives
1. Introduce the concepts of existence and uniqueness of solution of differential equations
2. Develop analytical techniques to solve differential equations
3. Understand the properties of solution of differential equations

Learning Outcomes
1. Understanding existence, uniqueness, and other properties of a solution of differential equations
2. Solving differential equations with series method, Green's function method
3. Solving systems of ODE, and characterization of their solutions

Contents
2. Second Order Linear Equations: General solution of homogeneous and non-homogeneous equations with constant coefficients, method of undetermined coefficients, method of variation of parameters, Sturm comparison theorem, Sturm separation theorem, Green's function method, variable coefficients equations, Series solution method, Legendre's equation, Frobenius method, Bessel's equation, Sturm-Liouville problems, orthogonal eigenfunction expansions
4. Applications of differential equations

Reference Books
Course Title: Probability and Statistics
Course No.: MA514
Department: Mathematics
Structure (LTPC): 3-0-0 [3]
Offered for: M.Sc. Students
Type: Compulsory
Pre-requisite: None

Objectives
1. Demonstrate an understanding of the basic principles of probability theory.
2. Use of the properties of discrete and continuous random variables with their joint, marginal, and conditional distributions.
3. Use of the various families of probability distributions to model various types of data.
4. Understanding of random sampling, theory of estimation and testing of hypotheses.

Learning Outcomes
Understanding of probability theory and statistics to solve industrial problems

Contents
1. Probability measure, conditional probability, Bayes’ theorem; Random variable, cumulative distribution function and its properties, probability density function, functions of a random variable, transformation, moments, standard discrete and continuous distributions and their applications, Chebychev’s inequality; Random vectors, joint, marginal and conditional distributions, conditional expectation, independence, correlation and regression.
2. Bi-variate normal distribution, functions of random vectors, transformation, Convergence of sequences of random variables, weak and strong laws of large numbers, central limit theorems, sampling distributions, estimation of parameters, maximum likelihood method and method of moments, interval estimation, testing of hypotheses, ANOVA
3. Applications of probability and statistics

Reference Books
Course Title     Programming Techniques  Course No.    MA515
Department       Mathematics       Structure (LTPC)  3-3-0 [4]
Offered for      M.Sc. Students    Type            Compulsory
Pre-requisite    None

Objectives
1. To introduce the basics of computer programming.
2. To understand and develop well-structured program in C/C++.

Learning Outcomes
1. Ability to implement programs using C/C++.
2. Ability to implement fundamental data structures in C/C++.

Course Contents
1. Introduction: The Von-Neumann architecture, machine language, assembly language, high level programming languages, compiler, interpreter, loader, linker, text editors and flowchart.
2. Basic features of programming (Using C/C++): Data types, variables, operators, expressions, control structures, functions, parameter passing conventions.
3. Advanced features of programming: Arrays, Multi-dimensional arrays, recursions, operation on data (Insert, delete, search, traverse and modify), structures, memory management, files, input/output, standard library functions, programming tools and pointers, introduction to object oriented programming.

Reference Books
Course Title: Abstract Algebra
Course No.: MA521
Department: Mathematics
Structure (LTPC): 3-0-0 [3]
Offered for: M.Sc. Students
Type: Compulsory

Objectives
1. To train the student in the domain of Abstract Algebra.
2. To give sufficient knowledge of the subject, which can be used by student for further applications in their respective domains of interest.

Learning Outcomes
1. Modules, Dual modules, Cyclic Primary modules and Decomposition theorem.
2. Nilpotent and Solvable groups, Galois Theory, splitting Fields, Normal extension, Fundamental theorem of Galois theory

Contents
1. Groups and Rings, Transformation Groups, Polynomial Rings, Principal Ideal Domain, Unique Factorization domain, Prime Fields and Euclidean Algorithm
2. Applications to Abelian groups, Sylow Theory, nilpotent and solvable groups, Galois Theory, splitting Fields, Normal extension, Fundamental Theorem of Galois Theory
3. Applications of algebra

Reference Books
Course Title: Complex Analysis  
Course No.: MA522  
Department: Mathematics  
Structure (LTPC): 3-0-0 [3]  
Offered for: M.Sc. Students  
Type: Compulsory  
Pre-requisite: None

Objectives
1. To train the student in the area of complex analysis
2. To give sufficient knowledge of the subject which can be used by student for further applications in their respective domains of interest

Learning Outcomes
1. Cauchy-Riemann Equations, Homotopy of paths,
2. Cauchy-Goursat theorem and Cauchy’s Integral Formula,
3. Evaluation of improper integrals

Contents:
1. Complex functions, continuity, differentiability, Cauchy Riemann Equations, elementary functions, Taylor and Laurent Series expansion, Contour Integration, anti-derivatives of a complex function, Cauchy-Goursat Theorem, Homotopy, Simply and multiply connected domains, Cauchy’s Integral formula, Liouville’s Theorem, Fundamental Theorem of Algebra, Maximum modulus principle, Residues and poles, Cauchy’s residue theorem, Jordan’s Theorem, evaluation of Improper integrals, conformal mappings.
2. Applications of complex analysis

Reference Books
**Objectives**
1. Introduce the concepts of existence and uniqueness of solution of differential equations
2. Develop analytical techniques to solve differential equations
3. Understand the properties of solution of differential equations

**Learning Outcomes**
1. Understanding existence, uniqueness, and other properties of a solution of differential equations
2. Solving differential equations with series method, Green's function method, Laplace transform method, and Fourier transform method

**Contents**
1. *Motivations and origins, PDEs of engineering and physics (Heat, wave and Laplace equations)*
2. *First order partial differential equations*: Linear, quasi-linear and fully nonlinear equations, Lagrange method, the Cauchy problem, Cauchy method of characteristics, compatible systems, Charpit methods
4. *Elliptic, parabolic and hyperbolic equations*: Laplace/Poisson equation, heat conduction equation, wave equation, Burger equation, Initial and/or boundary value problems, D’Alembert’s solution, Maximum-Minimum principles, Duhamel’s principle, solutions by Green’s function method
5. Applications of partial differential equations

**Reference Books**
Course Title: Numerical Analysis  
Course No.: MA524  
Department: Mathematics  
Offered for: M.Sc. Students  
Type: Compulsory

Objectives
1. Learn numerical techniques for variety of mathematical problems  
2. Analyze the validity and error in the numerical results

Learning Outcomes
1. Solve numerically algebraic equations, linear systems of equations, ordinary and partial differential equations, eigenvalue problems; Carry out numerical differentiation, integration and interpolation  
2. Develop understanding of numerical error and applicability of a particular method

Contents
1. Errors: Roundoff errors and truncation errors  
2. Root finding methods: the bisection method, the method of false position, fixed point iteration method, the Newton-Raphson method, the secant method, Muller’s method  
3. Numerical solution of linear system of equations: Direct methods (Gauss elimination, Gauss-Jordan method, LU decomposition), iterative methods (Gauss Seidel method, Gauss-Jacobi method), matrix inversion, the power method for eigenvalue problems  
4. Interpolation: Errors in polynomial interpolation, finite differences, Newton’s formulae for interpolation, central difference interpolation formulae, Lagrange interpolation formula, Hermite’s interpolation formula, Spline interpolation, cubic spline  
6. Numerical differentiation: finite difference method, high accuracy differentiation formulas, Richardson Extrapolation  
8. Applications of numerical methods

Tools to be used: MATLAB, Mathematica, Octave, Scilab

Reference Books
Course Title: **Functional Analysis**  
Course No.: **MA531**  
Department: **Mathematics**  
Structure (LTPC): **3-0-0 [3]**  
Offered for: **M.Sc. Students**  
Type: **Compulsory**  
Pre-requisite: **None**

**Objectives**
1. To introduce the area of Functional analysis.
2. To give sufficient knowledge of the subject, which can be used by student for further applications in their respective domains of interest.

**Learning Outcomes**
1. Basic idea of a normed linear spaces and operators on normed linear space.
2. Open Mapping theorem, Hahn-Banach Theorem and their applications.

**Contents**
3. Applications of functional analysis

**Reference Books**
Course Title: Optimization Techniques  
Course No.: MA525  
Department: Mathematics  
Offered for: M.Sc. Students  
Structure (LTPC): 3-0-0 [3]  
Type: Elective  
Pre-requisite: None

Objectives
1. To train the student in the domain of linear and non-linear programming
2. To give sufficient tools for solving programming problems which can be used by student for further applications in different areas of interest

Learning Outcomes
1. Simplex method, Charne’s M-method, Two-phase Method
2. Duality and Integer programming problem
3. Transportation problem, assignment Problem, Karmakar’s Algorithm and Wolf’s method
4. Newton’s method, Conjugate direction method and Quasi-Newton methods
5. KKT conditions, penalty and Barrier Methods, exact penalty functions

Contents
2. Nonlinear Programming: First and second order conditions, Iterative methods, line search methods, global convergence of descent algorithms, Newton’s method, Conjugate direction method, Quasi-Newton Method, Constrained optimization - Lagrange Multipliers, Karush-Kuhn-Tucker conditions, Regular points, Sensitivity analysis, Quadratic Programming, Convex problems
3. Applications of optimization

Reference Books
Objectives
1. To train the students in the domain of Topology
2. To give sufficient knowledge of the subject which can be used by student for further applications in their respective domains of interest

Learning Outcomes
1. Topological spaces, order topology and product topology
2. Separation axioms, countability axioms Urysohn’s metrization theorem

Contents
1. Topological spaces, basis and subbasis, order topology, product topology, subspace topology, convergence, closed sets, continuous functions, quotient topology, Connected spaces, Components, local connectedness, compact spaces, limit point compactness, local compactness
3. Applications of topology

Reference Books
<table>
<thead>
<tr>
<th>Course Title</th>
<th>Wavelet Analysis and Applications</th>
<th>Course No.</th>
<th>MA552</th>
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<td>Type</td>
<td>Elective</td>
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<tr>
<td>Pre-requisite</td>
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**Objectives**

1. To introduce the origin of wavelet transform
2. To develop the understanding of multiresolution analysis for different types of signals
3. To equip the students with various possible applications of the wavelet transform

**Learning Outcomes**

1. Difference between Fourier and wavelet transform.
2. Answer to the question why wavelet transform is a better tool for signal analysis.
3. Implement and apply wavelet transform for various applications.

**Contents**

1. *Fourier Analysis*: Fourier and inverse Fourier transforms, Gabor transform, short time Fourier transform and the uncertainty principle
2. *Wavelet Analysis*: mother wavelet, orthogonal and biorthogonal wavelets, continuous and discrete wavelet transform, basic properties of wavelet transforms, multiresolution analysis, Haar wavelet transform, Dabuchies Wavelet Transform, wave-packet transform
3. *Applications*: Application of wavelet theory to feature detection, image denoising and compression

**Reference Books**

Course Title: Complex Networks
Department: Mathematics
Offered for: M.Sc. Students
Pre-requisite: 

Structure (LTPC): 3-0-0 [3]
Type: Elective

Objectives
1. To educate and train the students into fundamentals of complex networks and its applications.
2. To provide hands on training along with illustrative examples of real-life systems.

Learning Outcomes
1. Understanding concepts of network and network phenomena in real life
2. Applications of network theory

Course Contents
1. Introduction to Graph Theory: Introduction to graph theory, Examples of graphs, Directed and undirected networks, Graph theoretical metrics, Degree distribution, Clustering, Adjacency matrix
2. Classical random graphs: Classical models, Loopholes in random graphs, Giant component
3. Small and large worlds: Diameter of the Web, Equilibrium versus growing tree, Fractal nature of giant connected component
5. Weighted Networks: The strength of weak ties, World-wide airport network, Airport network of India Modeling weighted networks
7. Applications of complex networks modeling: Examples of real-world networks

References
Objectives
1. To introduce the concept of linear and nonlinear dynamical systems
2. To learn the basic ideas and methods associated with dynamical systems, like, evolution of system, fixed points, periodic points, attractors, bifurcation process and stability of the systems
3. To understand the nonlinearity in nature and study of the nonlinear models in engineering and its dynamics
4. Use Matlab and simulink for solving dissipative dynamical systems which are more relevant to the engineering problems

Learning Outcomes
1. Construction of phase portraits of nonlinear system and understanding of fundamental difference between linear and nonlinear systems.
2. Identification of fixed points, periodic points and limits cycles and determine their stability.
3. Elementary bifurcations like, saddle node, period doubling etc.
4. Concept of attractors, chaotic attractors; measurement of chaos and its application to various engineering models.

Contents
1. History of Dynamics, The importance of being Nonlinear, A Dynamical view of the world, Examples of dynamical systems, Uncoupled Linear systems, Diagonalization, Exponential of operators, Linear systems in $R^2$ and Stability theory, Nonhomogeneous Linear systems. Nonlinear differential equations, Vector field of nonlinear systems, Phase portrait. Limit cycles and their stability
2. Logistic maps, period doubling bifurcations, Flip and tangent bifurcations, Periodic windows, Intermittency transcritical, Lyapunov exponent, Universality and Experiments, Renormalization, Cantor Set, Two dimensional maps, Bifurcation in two-dimensional maps
4. Lyapunov function and Central Manifold theory, Non-smooth bifurcations, bifurcation in piecewise smooth 2-dim map, multiple attractor bifurcation
5. Application of dynamical systems

Reference Books

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Course Booklet for M.Sc. (Mathematics)
2015