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Annexure 3

Syllabus for Ph.D. entrance examination

(1) **Linear Algebra**: Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, eigenvalues and eigenvectors, minimal polynomial, Cayley-Hamilton Theorem, diagonalization, Jordan canonical form, symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal and unitary matrices; Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, definite forms.

(2) **Algebra**: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Principle ideal domains, Euclidean domains, polynomial rings and irreducibility criteria; Fields, finite fields, field extensions.

(3) **Calculus**: Finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property; Sequences and series, convergence; Limits, continuity, uniform continuity, differentiability, mean value theorems; Riemann integration, Improper integrals; Functions of two or three variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications; Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

(4) **Complex Analysis**: Analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's theorem & Laurent's theorem; residue theorem and applications for evaluating real integrals; Rouche's theorem, Argument principle, Schwarz lemma; conformal mappings, bilinear transformations.

(5) **Real Analysis**: Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence; Weierstrass approximation theorem; Power series; Functions of several variables: Differentiation, contraction mapping principle, Inverse and Implicit function theorems; Lebesgue measure, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

(6) **Functional Analysis**: Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, Riesz representation theorem.

(7) **Numerical Analysis**: Numerical solutions of algebraic and transcendental equations: bisection, secant method, Newton-Raphson method, fixed point iteration; Interpolation: error of polynomial interpolation,Lagrange and Newton interpolations; Numerical differentiation; Numerical integration: Trapezoidal and Simpson's rules; Numerical solution of a system of linear equations:direct methods (Gauss elimination, LU decomposition), iterative methods (Jacobi & Gauss-Seidel); Numerical solution of initial value problems of ODEs: Euler's method, Runge-Kutta methods of order 2.

(8) **Ordinary Differential equations**: First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations.

(9) **Probability and Statistics:** Probability space, conditional probability, Bayes theorem, independence, Random variables, joint and conditional distributions, standard probability distributions and their properties, expectation, conditional expectation, moments; Weak and strong law of large numbers, central limit theorem; Sampling distributions, UMVU estimators, maximum likelihood estimators, Testing of hypotheses, standard parametric tests based on normal, X2 , t, F – distributions; Linear regression; Interval estimation.

(10) **Optimization:** Linear programming: Basic feasible solution, polyhedral set, extreme point and extreme direction, representation theorem of polyhedral set, simplex algorithm, duality. Nonlinear programming: Convex set, convex function, first order necessary condition for optimality, KKT condition. gradient descent algorithm, Lagrange multiplier. (Candidates opting for optimization should also prepare for Linear Algebra and Calculus.)

(11) **Discrete mathematics:** Propositional Logic, First Order Logic, Relations, Functions, Partial Orders, Lattice, Boolean Algebra, Asymptotic - Big O, Omega, and Theta notations, comparisons and operations on asymptotic classes, Recurrence relations, Connectivity in graphs: paths, connected components, Trees and their properties, tree traversals, minimum spanning tree, Vertex and Edge covering, Matching, Independent sets, Vertex coloring.