Curriculum

Ph.D.

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Indian Institute of Technology Jodhpur

Ph.D. (Mathematics)

Cat.	Course Number: Course Title	L-T-P	Credits		Cat.	Course Number: Course Title	L-T-P	Credits
I Semester					**II Semester**			
E	*Electives*				E	*Electives*		
	Total					*Total*		
III Semester					**IV Semester**			
H	**MA799 Ph.D. Thesis**				H	**MA799 Ph.D. Thesis**		
	Total					*Total*		
V Semester					**VI Semester**			
H	**MA799 Ph.D. Thesis**				H	**MA799 Ph.D. Thesis**		
	Total					*Total*		
VII Semester					**VIII Semester**			
H	**MA799 Ph.D. Thesis**				H	**MA799 Ph.D. Thesis**		
	Total					*Total*		

Electives

Semester I				Semester II			
MA751	Applied Linear Algebra	3-0-0	3	MA753	Optimization for Systems	3-0-0	3
MA752	Differential Equations for Systems Modelling	3-0-0	3	MA757	Numerical Partial Differential Equations	3-0-0	3
MA754	Algorithmic Graph Theory	3-0-0	3	MA758	Stochastic Processes	3-0-0	3
MA755	Chaos Theory and its Applications	3-0-0	3				
MA756	Functional Analysis	3-0-0	3				

S.No.	*Category*	*Category Title*	Students with	*Total Courses*	*Total Credits*
1	E	*ELECTIVES*	Master's Degree	4	12
			Bachelor's Degree	10	30
2	H	Thesis	-	-	-

Course Title	***Applied Linear Algebra***	Course No.	**MA751**			
Department	System Science	Structure	3	0	0	3
Offered for	PhD	Status			*Elective*	
Pre-requisite	Consent of the Teacher	To take effect from				

Objectives

This course introduces students to key linear algebraic notions and methods that are fundamental to a variety of problems in systems modelling and analysis. The course will provide insight in the underlying concepts and in the practical usability of the methods

Learning Outcomes

1. Direct and iterative Methods for large scale linear system solving and eigenpair computation.
2. Matrix methods in data analysis.
3. Application of linear algebra in linear dynamical systems.

Contents

Gaussian elimination methods for solving linear system, sparse Gaussian elimination, LU decomposition, positive definite matrices, Cholesky decomposition, Sensitivity analysis of linear systems, Preconditioners, Descent Methods (Steepest Descent), The Conjugate-Gradient Method, The least squares method, Singular value decomposition, QR decomposition, Eigenvalues and eigenvectors, Power method, The QR algorithm

Basics of linear dynamical systems, Stability of linear systems, Matrix exponentials, Controllability and observability of linear systems

Reference Books:

1. Meyer, Carl D. *Matrix analysis and applied linear algebra*. Vol. 2. SIAM, 2000.
2. Trefethen, Lloyd N., and David Bau III. *Numerical linear algebra*. Vol. 50. SIAM, 1997.
3. Golub, Gene H., and Charles F. Van Loan. *Matrix computations*. Vol. 3. JHU Press, 2012.
4. Watkins, David S. *Fundamentals of matrix computations*. Vol. 64. John Wiley & Sons, 2004.
5. Kailath, Thomas. *Linear systems*. Vol. 1. Englewood Cliffs, NJ: Prentice-Hall, 1980.

Course Title	***Differential Equations for System's Modelling***	Course No.	**MA752**			
Focus Group	Mathematics	Structure	3	0	0	3
Offered for	PhD	Status			*Elective*	
Pre-requisite	Basics of ODEs, PDEs, and linear algebra	To take effect from				

Objectives

This course discusses various analytical, numerical and approximation techniques of solving ordinary and partial differential equations arising out of modeling of real life systems.

Learning Outcomes

1. Solve systems of differential equations
2. Solve nonlinear equations numerically
3. Analyse nature of the solution of differential equation by perturbational and asymptotic method
4.

Contents

Local existence of solutions, uniqueness of solutions, dependence of the solution on the initial conditions, comparison theorems, methods for solving initial value problems and boundary value problems involving ODEs

Systems of first order linear constant coefficient ODEs: converting to systems of first-order differential equations, linearly independent and Wronskian, full set of real eigenvectors, complex eigenvectors, generalized eigenvectors, diagonalization, the matrix exponential, nonhomogeneous systems of first-order equations, application to predictor-prey models, population dynamics

Partial differential equations: Fourier series method, Laplace equation, heat conduction equation and wave equation, application to traffic flow problem

Numerical computation environments: MATLAB, Octave, Scilab, Mathematica

Numerical methods: Taylor series methods, Runge–Kutta methods, error analysis, numerical methods for higher-order systems, stiff systems, finite difference method for partial differential equations, numerical experiments

Nonlinear systems: multiple equilibria and chaos, the phase plane, Poincare sections, limit cycles, linearization, Jacobian linearization, geometry and stability of equilibrium points in the phase plane, harmonic balance, describing functions, saddle-node bifurcations, pitchfork bifurcations

Perturbational method and asymptotic methods for solution of nonlinear equations

Reference Books:

1. A.C. King, J. Billingham and S.R. Otto, Differential Equations: Linear, Nonlinear, Ordinary, Partial, Cambridge University Press, 2003.
2. R. C. McOwen, Partial Differential Equations: Methods and Applications, 2nd Ed., Pearson Education, Inc., 2003.
3. B. Goodwine, Engineering Differential Equations: Theory and Applications, Springer Publication, 2010.

Course Title	***Optimization for Systems***	Course No.	**MA753**			
Focus Group	Mathematics	Structure	3	0	0	3
Offered for	PhD	Status				*Elective*
Pre-requisite	Consent of Teacher	To take effect from				

Objectives

The main objective of this course includes the development of mathematical methods and numerical algorithms to solve convex and non-convex optimization problem of different patterns. The derivation of different optimization problems associated with network systems, financial systems, telecommunication systems, robotics etc will be illustrated

Learning Outcomes

1. Understanding importance of learning solution techniques for optimization problems
2. Understanding the importance of structure/pattern of a problem
3. Understanding of real-life decision problems modelling and solution
4. Development of analytical thinking through Mathematical sophistication

Contents

Basic notions of unconstrained/global optimization problems: Optimality conditions, Gradient methods and its convergence, Newton's method and variations.

Convex optimization: Basics of convex analysis, Linear programming, Conic programming, Conic quadratic programming, Semidefinite programming, Interior-point polynomial time methods for convex problems.

Non-convex optimization: Computational tractability of convex programs, Conjugate-Gradient methods for non-convex problems, Preconditioned conjugate-gradient methods.

Applications to systems: Derivation and solution of optimization problems arise in network systems, financial systems, telecommunication systems, robotics etc.

Reference Books:

1. S.P. Boyd and L. Vandenberghe. Convex Optimization. Cambridge University Press, 2004
2. D. Bertsekas. Nonlinear Programming. Athena Scientific, Massachusetts, 1999
3. R. Pytlak. Conjugate-Gradient Algorithms in nonconvex optimization. Springer, 2009
4. G. Cornuejols and R. Tütüncü. Optimization Methods in Finance. Cambridge University Press, 2007
5. D. P. Bertsekas. Linear Network Optimization: Algorithms and Codes. MIT press, 2003
6. Jon Dattorro. Convex Optimization & Euclidean Distance Geometry. Mεβoo Publishing, 2011

Course Title	***Algorithmic Graph Theory***	Course No.	**MA754**			
Department	Mathematics	Structure (LTPC)	3	0	0	3
Offered for	Ph.D.	Status			Elective	
Pre-requisite	Discrete Mathematics, Graph Theory	To take effect from	July 2014			

Objectives

1. To introduce the concepts of special classes of graphs
2. To give idea about "P vs NP" for graph optimization problems

Learning Outcomes

1. Ability to know the how structural characterizations of graphs help in designing efficient algorithms for graph optimization problems

Course Content

Introduction to graph problems: Definition of different graph parameters such vertex cover, edge cover, longest path, maximum clique, maximum independent set, maximum matching, graph coloring, and the optimization problems associated with those. A survey on computational complexity of solving above problems for arbitrary graphs

Interval Graphs and Chordal Graphs: Intersection graphs: Definition and properties, Examples, Definition and Characterization of interval and chordal graphs, On solving different graph problems on these classes along with NP-hard problems on these graph classes

Subclasses of Bipartite Graphs: Definition and characterization of some important subclasses of bipartite graphs and solving optimization problems on these classes

Other classes of graphs: Split graphs, cographs, threshold graphs

Reference Books:

1. A. Brandstadt, V. B. Le and J. P. Spinrad, Graph Classes: A survey, SIAM Monograph, 1987
2. M. C. Golumbic, Algorithmic graph theory and perfect graphs, Elsevier, 2004
3. J. P. Spinrad, Efficient Graph Representations, Field Institute Monograph, AMS, 2003
4. D. B. West, Introduction to Graph Theory, 2nd Edition, PHI Learning, 2009

Course Title	**Chaos Theory and its Applications**	Course No.	**MA755**			
Focus Group	Mathematics	Structure (LTPC)	3	0	0	3
Offered for	PhD	Status			Elective √	
Pre-requisite		To take effect from	July 2014			

Objectives

1. To equip the students with the basic fundamentals of chaos theory.
2. Learning and understanding various routes to chaos
3. To introduce applications of chaos to various physical systems and its introduction in IFS.

Learning Outcomes

1. Notion of chaos in one dimension.
2. Understanding Bifurcation theory and different routes to chaos.
3. Two dimensional flows, phase portraits and chaos in higher dimension.

Course Content

One dimensional maps, Fixed points, periodic points and Stability, Basin of Attraction, Bifurcation, period doubling and route to chaos, Sharkovsky's theorem, Li-Yorke Theorem Lorentz System and chaos.

Chaos in one dimension, density of periodic points, transitivity, sensitive dependence on initial conditions, cantor's set, conjugacy, symbolic dynamics, Rossler's Attractor. Various notions of chaos.

Stability of two dimensional Maps, Phase space diagrams, Linear Systems and their stability, Hartman-Grobman Theorem, Stable Manifold Theorem, Bifurcation and chaos in two dimensions.

One dimensional flows, chaos in one dimension, Fixed points and Stability, Flows on the circle, uniform oscillator, non-uniform oscillator, overdamped pendulum, Comparison with one dimensional flows.

Bifurcation, Saddle node Bifurcation, Transcritical Bifurcation, Laser Threshold, Pitchfork Bifurcation, Hopf Bifurcation.

Two dimensional flows, definition and examples, phase portraits, existence and uniqueness, fixed points and linearization, conservative systems, reversible systems, limit cycles, Poincare-Bendixson Theorem, chaos in two dimensions

Fractals, countable and uncountable sets, cantor set, dimension of self similar fractals, Box dimension, pointwise and correlation dimension, Iterated Function System, Collage Theorem and Image Compression, Julia and Mandelbrot Sets.

Reference Books:

1. Saber N. Elaydi, Discrete Chaos: With Applications in Science and Engineering, Second Edition.
2. Steven H. Strogatz, Nonlinear Dynamics and Chaos : With Applications to Physics, Biology, Chemistry and Engineering.
3. H Nagashima and Y Baba, Introduction to Chaos: Physics and Mathematics of Chaotic Phenomena.

Course Title	**Functional Analysis**	Course No.	**MA756**			
Focus Group	Mathematics	Structure (LTPC)	3	0	0	3
Offered for	PhD	Status			Elective	
Pre-requisite	Consent of Teacher	To take effect from	July 2014			

Objectives:
The objective of the course is to train the students in the area of Functional analysis. In this course, students will be introduced to areas like Banach algebras and Spectral theory. Bounded operators on Hilbert space, spectral theorem, positive operators and square roots are some of the advanced topics offered in this course.

Learning Outcomes:
Students trained in this area and having a good understanding of the subject and are expected to successfully apply in the domain of its applications. At the end of the course, student is expected to be have good knowledge of topics like Banach algebras, complex homomorphisms, spectral theory, bounded and unbounded operators on Hilbert spaces which can be applied in their respective areas.

Course Content
1. Normed linear space, Banach space, subspace of a normed linear space.
2. Linear operator, Heine-Borel property, Equivalent norms, norm of a linear operator, boundedness and continuity, quotient spaces.
3. Measurable and Integrable functions, Lp spaces, Minkowski's inequality, Holder's inequality, separability of Lp spaces, uniformly bounded operator, principle of uniform boundedness.
4. Sub-linear functional, Semi-norm, Open mapping theorem, Closed graph theorem, Hahn Banach Theorem, dual of a normed linear space, reflexive normed linear space.
5. Inner product spaces, polarisation identity, Bessel's inequality, Hilbert space, Riesz representation theorem.
6. Introduction to Banach Algebras, Complex homomorphisms, Basic properties of spectra, Symbolic Calculus, Differentiation, Group of invertible elements.
7. Bounded operators on Hilbert space, resolutions of the identity, spectral theorem, Eigenvalues of Normal Operators, positive operators and square roots.
8. Unbounded operators, Graphs and symmetric operators, Cayley's Transform, Resolutions of identity, spectral Theorem.

Reference Books:
1. Rudin, W. Functional Analysis, McGraw-Hill Publisher, 1991. ISBN No. 0070542368
2. Aliprantis, D. Charalambos, Burkinshaw, O. Principles of Real Analysis, Academic Press, 1998. ISBN No. 0120502577
3. Kreyszig, E. Introductory Functional Analysis with applications, Wiley, 1989, ISBN No. 0471504599

Course Title	**Numerical Partial Differential Equations**	Course No.	**MA757**			
Focus Group	Mathematics	Structure (LTPC)	3	0	0	3
Offered for	PhD	Status			Elective	
Pre-requisite	Consent of Teacher	To take effect from	July 2014			

Objectives

1. Numerical solution of partial differential equations by finite difference methods

Learning Outcomes

1. Application of finite difference method for numerical solution of partial differential equations
2. Error analysis and stability analysis of finite difference schemes

Course Contents

Finite difference methods for elliptic, parabolic and hyperbolic problems, stability, consistency and convergence theory, dissipation and dispersion, error estimates, conservation laws and the energy method of analysis.

Elliptic equations: Stability analysis by the energy method, Error analysis with a maximum principle, asymptotic error estimates, convergence result.

Parabolic equations: Fourier analysis of the error, Alternating direction implicit method.

Hyperbolic equations: CFL condition, Von Neumann stability Analysis, upwind scheme, Lax-Wendroff scheme, leap-frog scheme, Hamiltonian systems and symplectic schemes.

Numerical linear algebra: Classical iterative Methods for solving linear systems, Fourier analysis of convergence, large sparse linear systems, GMRES algorithm, preconditioned Conjugate Gradient method, multigrid method, Newton's method and its variations for solving nonlinear systems.

Reference Books

1. J. W. Thomas, Numerical partial differential equations: Finite difference methods, Springer, 1995.
2. Quarteroni, A. and Valli, A. Numerical Approximation of Partial Differential Equations, Springer, 1997.
3. K. W. Morton and D. F. Mayers, Numerical solution of partial differential equations, CUP, 2005.
4. L. N. Trefethen, and David Bau III, Numerical Linear Algebra, SIAM, 1997.

Course Title	**Stochastic Processes**	Course No.	**MA758**			
Focus Group	Mathematics	Structure (LTPC)	3	0	0	3
Offered for	PhD	Status			Elective	
Pre-requisite	Consent of Teacher	To take effect from	July 2014			

Objectives

1. Familiar with the basic concepts of the theory of stochastic processes in continuous time
2. To be able to use various analytical and computational techniques to study stochastic models that appear in applications

Learning Outcomes

1. Understand the notion of a stochastic, and how simple ideas of stochastic processes can be used in real life
2. To develop skills in building stochastic models using Markov chains
3. Handling stochastic differential equations

Course Contents

Probability theory and random variables: Basic definitions, probability spaces, probability measures etc. Random variables, conditional expectation, characteristic functions, limits theorems

Stochastic processes: Basic definitions. Brownian motion. Stationary processes. Other examples of stationary processes. The Karhunen-Loeve expansion

Markov processes: Introduction and examples. Basic definitions. The Chapman-Kolmogorov equation. The generator of a Markov process and its adjoint. Ergodic and stationary Markov processes

Diffusion processes: Basic definitions and examples. The backward and forward (Fokker-Planck) Kolmogorov equations. Connection between diffusion processes and stochastic differential equations

The Fokker-Planck equation: Basic properties of the FP equation. Examples of diffusion processes and of the FP equation. The Ornstein-Uhlenbeck process. Gradient flows and eigenfunction expansions

Stochastic Differential Equations: Basic properties of SDEs. Itô's formula. Numerical solution of SDEs

Reference Books

1.Grimmett and Stirzaker: Probability and Random Processes, 3rd Edition, Oxford University Press
2.Karlin and Taylor: A First Course in Stochastic Processes, 2nd Edition, Academic Press, 1975
3.Lawler: Introduction to Stochastic Processes, 2nd Edition, Chapman and Hall, 2006
4.Resnick: Adventures in Stochastic Proceses, BirkHauser, 1992